Controlling the Basin of Attraction of Period-1 Rotation of a Horizontally Excited Parametric Pendulum

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The equation of motion of a horizontally excited parametric pendulum is

$$\ddot{\theta} + c\dot{\theta} + a\sin\theta - b\cos t\cos\theta = 0,$$

which has no fixed points and has period-1 solutions as the simplest solution. Depending on the choice of parameters $a$, $b$ and $c$, Eq. (1) can have two qualitatively different periodic solutions viz. oscillation and rotation. There is a growing interest in rotating solutions as they can be used for energy harvesting. For small values of $b$, only oscillatory solutions exist and rotation appears through a saddle-node bifurcation with increasing $b$. In general, period-1 rotation always coexists with some oscillatory solution and the basin of attraction of rotation does not extend to the entire initial condition space. For example, at $a = 0.5$, $b = 0.1$ and $c = 0.03$, a stable period-1 oscillation coexists with period-1 rotation reducing its basin of attraction (see fig. 1).

Our goal is to ensure robust initiation of period-1 rotation from all initial conditions. We employ a delayed control, an extension of Pyragas’ control [1] to period-1 rotation, as

$$\ddot{\theta} + c\dot{\theta} + a\sin\theta - b\cos t\cos\theta = K[\theta(t-2\pi) - \theta(t) + 2\pi\Lambda],$$

where $\Lambda = Round\left[\frac{\theta(t) - \theta(t-2\pi)}{2\pi}\right]$ (rounded to nearest integer) and $K$ is the control gain. For stability studies, we linearize about $\Theta = \{\text{period-1 oscillation and rotation}\}$ to get

$$\ddot{\eta} + c\dot{\eta} + (a\cos\Theta + b\cos t\sin\Theta)\eta = K[\eta(t-2\pi) - \eta(t)],$$

The dominant Floquet multipliers as function of control gain ($K$) corresponding to period-1 attractors are calculated by applying semi-discretization method [2] to Eq. (3). To get an analytical expression for the periodic solutions (required to calculate the Floquet multipliers), we use the harmonic balance method [3]. From the variation of the dominant Floquet multiplier with $K$ (see fig. 2), we observe that for $K \in [0.0220, 0.0279]$, period-1 oscillation is unstable but period-1 rotation remains stable. However, the basin of attraction of rotation for any value of $K$ in this range does not fill the entire initial condition space since we get a stable quasi-periodic oscillatory solution. We perform a bifurcation study of the various oscillatory attractors to identify the control gain $K$ at which all oscillatory attractors disappear. This information is then used to design a a delayed control with switching of the control gain between two values (one to destabilize all oscillatory attractors) while the other to stabilize the period-1 rotation. Details of these results will be presented at the conference.

Figure 1: Basins of attraction of rotations. Set of parameters: $a = 0.5$, $b = 0.1$ and $c = 0.03$.

Figure 2: Dominant Floquet multiplier. Set of parameters: $a = 0.5$, $b = 0.1$ and $c = 0.03$.

References