Quasiperiodic Bifurcation in 3D Piece-wise Linear maps

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Much research has been reported on the bifurcations of periodic solutions in both in continuous and discrete dynamical systems [1, 2]. Bifurcations in quasiperiodic solutions have also been investigated both in discrete and continuous time smooth dynamical systems [3, 4]. Local bifurcations that can occur in a quasiperiodic orbit can be classified in four ways: (a) a torus doubling resulting in two disjoint loops, (b) a torus doubling resulting in a single closed curve with double length, (c) the appearance of a third frequency, (d) the birth of a stable torus and an unstable torus.

In this work we address the question: how does a quasiperiodic orbit bifurcate in a piecewise smooth system and how does it differ from the mechanisms described for smooth systems. We address this question with reference to the three dimensional piecewise normal form map:

$$X_{n+1} = F_\mu(X_n) = \begin{cases} A_l X_n + \mu C, & \text{if } x_n \leq 0 \\ A_r X_n + \mu C, & \text{if } x_n \geq 0 \end{cases}$$

where $X_n = (x_n, y_n, z_n)^T \in R^3$, $C = (1, 0, 0)^T \in R^3$, $A_l$ and $A_r$ are real valued $3 \times 3$ matrices

$$A_l = \begin{pmatrix} \tau_l & 1 & 0 \\ -\sigma_l & 0 & 1 \\ \delta_l & 0 & 0 \end{pmatrix} \quad \text{and} \quad A_r = \begin{pmatrix} \tau_r & 1 & 0 \\ -\sigma_r & 0 & 1 \\ \delta_r & 0 & 0 \end{pmatrix}$$

The phase space of this map is divided by the borderline $Sm : x = 0$ into two regions $L := (x, y, z) \in R^3 : x \leq 0$ and $R := (x, y, z) \in R^3 : x > 0$.

We have found four mechanisms of transitions from an invariant closed loop to different loop structures as depicted in Fig. 1. The bifurcation diagrams corresponding to the first two are shown in Fig. 2. In the paper we explain the mechanisms behind these transitions.

References


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