Controlling birhythmicity via conjugate self-feedback: Theory and experiment

Debabrata Biswas, Jürgen Kurths and Tanmoy Banerjee *

Bistability is a frequent variant of the commonly occurring dynamical feature, multistability, in many natural systems [1, 2, 3]. The most intriguing form of bistability is the the birhythmicity, i.e., the coexistence of two stable limit cycles of different amplitude and frequency. This type of oscillations is very common, particularly, in physics (e.g., energy harvesting system, see [4] and references therein) biology (e.g. glycolytic oscillator and enzymatic reactions [1, 2]) and chemistry [5]. Most of the biochemical oscillations that govern the organization of cell cycle, brain dynamics or chemical oscillations are birhythmic; examples include, birhythmicity in the p53-Mdm2 network, which is the key protein module that controls proliferation of abnormal cells in mammals [6], intracellular Ca2+ oscillations [2], oscillatory generation of cyclic AMP (cAMP) during the aggregation of the slime mold Dictyostelium discoideum [7] and circadian oscillations of the PER and TIM proteins in Drosophila [1].

In physical and engineering systems birhythmicity plays a negative role in limiting the efficiency of a certain application, e.g., in wind-induced electrical energy harvesting systems [4]) the efficiency demands the system to perform in the larger limit cycle. Further, the presence of birhythmicity makes a system vulnerable to noise: depending upon the noise intensity the system may end up in any of the two limit cycles [3]. Therefore, monorhythmicity is of practical importance in most of the physical systems. On the other hand, in networks of neuronal oscillators the occurrence of birhythmicity is often desirable to generate and maintain different modes of oscillations that organize various biochemical processes in response to variations in their environment [5]. Therefore, identifying an efficient control technique is of importance that can tame birhythmicity to yield monorhythmic oscillation or can retain its character intact where ever needed. However, apart from the study of controlling birhythmic behavior under time-delayed control technique by Ghosh et al. [8] no efficient control technique has been reported.

Being motivated by these facts we propose a technique to control the birhythmicity by introducing a self-feedback mechanism that incorporates the variable to be controlled and its canonical conjugate. Using a detailed analytical treatment, bifurcation analysis and experimental demonstrations we establish that the proposed technique is capable of eliminating birhythmicity and generates monorhythmic oscillation. The representative numerical and experimental time series and phase plane plots are shown in Fig. 1. Further, the detailed parameter space study reveals that, apart from monorhythmicity, the system shows transition between birhythmicity and other dynamical forms of bistability. We believe that our study may have potential applications in controlling birhythmicity in several mechanical and biochemical processes as well as in other fields.

Figure 1: Representative time series and phase plane plots of the birhythmic system under the proposed control scheme. (Left) Numerical results, (Right) Experimental results. From upper to lower panel: Transition from monobi- to monorhythmic oscillation.

References


*D. Biswas and T. Banerjee are with the Chaos and Complex Systems Research Laboratory, Department of Physics, University of Burdwan, Burdwan 713 104, West Bengal, India. Jürgen Kurths is with the Potsdam Institute for Climate Impact Research, Telegrafenweg, D-14415 Potsdam, Germany.