Multiscroll in coupled Lorenz oscillators

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We report here an interesting multiscroll dynamics in two undirectionally coupled Lorenz systems. The driver Lorenz system parametersa are set for the usual butterfly type attractor (2-scroll) while the response system is in resting state when uncoupled. We observed multiscroll attractors (3-, 4-, 5-, 6-scroll) at the response system in the weaker coupling regime. The multiscroll is seen as one after another additional scroll emerges in phase space of the response system with gradual decrease in coupling strength. The 3D attractors and their corresponding 2D phase portraits are shown in Fig.1. It appears as if a hidden multiscroll structure unfolds in the response oscillator, which is otherwise dormant in uncoupled state, when a butterfly type Lorenz attractor is forced into it and the forcing strength is weakened. The important practical applications of multiscroll are known as broadband signal generator and true pseudorandom number generator for communication engineering.

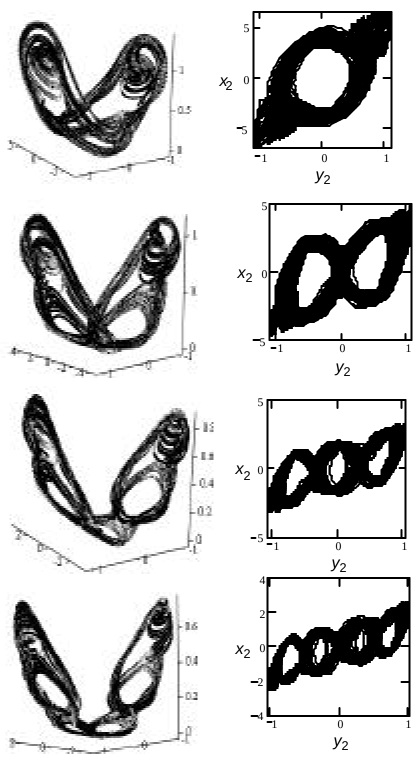

Fig.1. Numerical multiscroll in coupled Lorenz system: 3D attractors in left column; 3-scroll in the upper row for coupling strength, ε=2.85, 4-scroll in the second row for coupling strength, ε=1.9, 5-scroll in the third row for ε=1.313, 6-scroll for ε=0.95 in lowest row. The 2D phase portraits in the right column. Lorenz system parameters, b=8/3, σ=10 for both driver and response, r_1=35 for driver, r_2=1.15 for response.

The governing equations of the coupled Lorenz system is given by

$$\frac{dx_1}{dt} = s\,(y_1 - x_1) \quad (1a) \qquad \frac{dx_2}{dt} = s\,(y_2 - x_2) + e(x_1 - x_2) \quad (1d)$$

$$\frac{dy_1}{dt} = r_1 x_1 - y_1 - x_1 z_1 \quad (1b) \qquad \frac{dy_2}{dt} = r_2 x_2 - y_2 - x_2 z_2 \quad (1e)$$

$$\frac{dz_1}{dt} = -b z_1 + x_1 y_1 \quad (1c) \qquad \frac{dz_2}{dt} = -b z_2 + x_2 y_2 \quad (1f)$$

where eqns.1(a)-(d) represent the driver and eqns.1(d)-(e) represent the response Lorenz system. Parameters are b=8/3 and σ=10 for both the driver and the response while r_1=35 and r_2=1.15. The driver shows butterfly type attractor (2-scroll) for our choice of parameters while the response is in resting state when uncoupled.

We verified the results in experiments using electronic analog of Lorenz circuit. The circuit of two coupled Lorenz systems is shown in Fig.2 and the oscilloscope XY plots of multiscroll are shown in Fig.3 for different coupling.

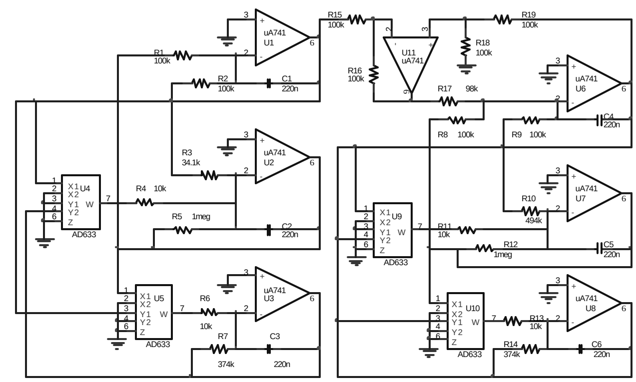

Fig.2. Circuit of two coupled Lorenz system.

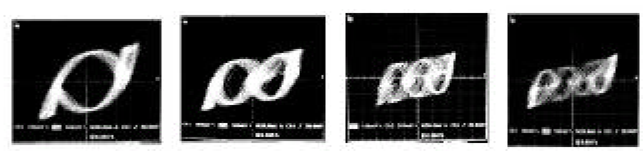

Fig.3. Experimental multiscroll in coupled Lorenz system: oscilloscope XY plots, 2D phase portraits from left, first 3-scroll for stronger R_{17}=98kΩ, second 4-scroll for R_{17}=264.2kΩ, third 5-scroll for R_{17}=375.5kΩ, right 6-scroll for R_{17}=437.8kΩ. Used Tektronix TDS 3014B oscilloscope (100MHz, sampling speed 1.25GS/s, data record length 10k).

We explained the origin of multiscroll using eignenvalue analysis and a bifurcation diagram which are not presented here due to page limitations. We also elucidated the origin of the multiscrolls using a schematic diagram, details of which are to be reported elsewhere.

References

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[3] S.K.Dana, B.K. Singh, S.Chakraborty, J.Kurths, G.Osipov, R.C.Yadav, P.K.Roy, C.-K.Hu, Multiscroll in coupled double scroll type systems (*to be reported elsewhere*).