Synchronization of Spatiotemporal Chaos and Its Application to Real-Time Secure Communication

S. Arora, M.S. Santhanam

Abstract: An approach to synchronize spatiotemporal chaos in one-way coupled map lattices has been proposed in this paper. It is shown that for sufficiently large coupling the spatiotemporal chaos in two identical one-way coupled map lattices, starting from different initial conditions, can be entirely synchronized using a common drive key obtained from a random number generator. We discuss the application of this phenomenon to real-time, secure, multi-channel communication networks and point out the advantages compared to similar earlier attempts.

Index Terms: Coupled Map Lattice, Spatiotemporal Chaos, Synchronization, Multi-channel Communication.

I. INTRODUCTION

In the last few years, several researchers have focused their attention on the problems related to chaos control and synchronization. The reason for this growing interest is due to the potential applications of chaos control, synchronization and dynamics in various areas ranging from secure communication [1-2], neural networks [3] and pattern formation etc.

The fact that coupling between the drive and the response subsystems can lead to synchronization was first discovered by Pecora and Carroll [4]. For instance, a well-known scheme [5] consists of taking a chaotic system, duplicating some subsystem and driving the duplicate and original subsystems with signals from the unduplicated part.

The information exchange operations can be performed simultaneously in parallel if one can properly drive and control the chaotic system with many channels using one key. This leads to enhanced efficiency of information handling as well as reduced hardware for such a purpose. A communication system resulting from such synchronization would not only be robust, but also be safer against external attacks and imitations since a high dimensional chaotic signal serves to mask the information content.

Synchronization of coupled map lattice is one approach that leads to applications in spread spectrum communications. An information signal consisting a message can be transmitted by using a chaotic signal as broadband carrier, of which the spectrum is much wider than that of information signal, and synchronization of chaos can be used to recover back the original signal. In spread spectrum communication, a single lattice site can be used as a channel to spread the signal at the transmitter, while a similar channel at the receiver, which is synchronized with the transmitter through a common drive signal, is used to despread the spreaded signal.

In this paper, we propose driving two one-way coupled map lattice (OCOML) [6-7], by a common drive key from a random number generator. We will also focus on dependence of lattice sites on initial conditions, synchronization error and number of iterations needed by a lattice site to come in sync with another lattice site.

The paper is arranged as follows. In section 2, procedure for the application of the drive key to the two one-way coupled map lattices is shown. In section 3, the above method is applied to synchronize the OCOML systems, and a study of the observed results is done. In section 4, we apply the above obtained results for signal masking and demasking. In section 5, we conclude by summarizing the observations and further discussing the applications of spatio-temporal chaos with respect to the observed results.

II. PROCEDURE

Recently, the researchers have studied the collective behavior of systems consisting of a large number of coupled identical units. Such models may represent active networks with elements located in the junctions of a discrete space lattice. In this paper we consider one-way coupled map lattice modeled as,

\[ X_{n+1}(i) = (1-\varepsilon) f[X_n(i)] + \varepsilon f[X_n(i-1)], \]

where \( i=1,2,3...L \) are the lattice sites, \( L \) is the system size, \( n = 0,1,2... \) corresponds to the iteration number and \( \varepsilon \) is the coupling strength between the adjacent lattices of the one-way coupled map lattice. The coupling strength \( \varepsilon \) is chosen to be sufficiently high to attain synchronization. We take \( f(x) = ax (1-x) \), where \( a \) is the logistic map parameter and \( X_0 \) is the initial condition.

Consider two one-way coupled map lattice (OCOML) systems with different initial conditions, where one is \( X(i) \) system and another is a \( Y(i) \) system. The lattice length is taken to be \( L=100 \), \( \varepsilon =0.9 \), while the initial conditions are prepared as pseudorandom numbers uniformly distributed in the interval \([0,1]\). In particular, we investigate the results of

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S. Arora is a final year (B.Tech) student at Dhirubhai Ambani Institute of Information and Communication Technology, Gandhinagar, Gujarat. email: siddharth_arora@da-iict.org.

M.S. Santhanam is in Theoretical Physics and Complex Systems Division at Physical Research Laboratory, Ahmedabad 380 009, Gujarat. e-mail: santh@prl.ernet.in.
driving both the systems by a common drive key, denoted by \( K_n \). So we have \( X_n (0) = K_n \), and an identical OCOML system \( Y (i) \) having different initial conditions as compared to \( X (i) \), but driven by the same key \( K_n \), i.e., \( Y_n (0) = K_n \). Thus we have the two systems as,

\[
\begin{align*}
X_n (0) &= K_n \\
X_{n+1} (i) &= (1-\varepsilon)f[X_n (i)] + \varepsilon f[X_n (i-1)], \\
i &= 1,2,...L
\end{align*}
\]

where \( K_n \) is randomly chosen sequence such that every element in \( K_n \) lies in the range \([0,1]\), and \( X_n (0) \) and \( Y_n (0) \) take this value \( K_n \) for all values of \( n \). Thus, we have

\[
\begin{align*}
Y_n (0) &= K_n \\
Y_{n+1} (i) &= (1-\varepsilon)f[Y_n (i)] + \varepsilon f[Y_n (i-1)], \\
i &= 1,2,...L
\end{align*}
\]

The advantage of our scheme as compared with other methods is that there is no need for a separate drive key generator. Previously such schemes \([8]\) used the output from a single lattice of a third CML as the drive key generator;

\[
\begin{align*}
Z_{n+1} (i) &= (1-\varepsilon)f[Z_n (i)] + \varepsilon f[Z_n (i-1)], \\
Z_n (i+L) &= Z_n (i), \\
i &= 1,2,...L
\end{align*}
\]

where \( Z (i) \) is a one way coupled ring map lattice system (OCRML). A randomly chosen sequence \( Z_n (i) \) forms the drive key \( K_n \), which is then equated to \( X_n (i) \) and \( Y_n (i) \) to drive the two systems. We rather choose a random sequence \( Z_n \) and allot it to the lattice, thus completely eliminating the need for a drive generator. Instead of first generating a matrix of \((n \times L)\) numbers using the OCRML in (3) and then choosing a sequence of only \( n \) numbers out of it, we obtain a sequence from random number generator and allot it to the one way coupled map lattice for all \( n \). Eliminating the need for a drive generator not only reduces the time for channel set up in real time communication, but also reduces the hardware complexity by removing any circuitry that would have been needed for the key generator \( Z_n \). Note that the demands on time in real time communication is more stringent than in offline communication.

We take two one-way coupled map lattices having different initial conditions, but driven by a common drive key. It can be analytically proved that for sufficiently high coupling strength \( \varepsilon \), the two OCOML systems can be synchronized. It is observed that \(|Y_n (i) - X_n (i)|\) monotonically decreases to zero, i.e., \(|Y_n (1) - X_n (1)| \rightarrow 0\), \(|Y_n (2) - X_n (2)| \rightarrow 0\)...as \( n \) increases. Two important requisites for synchronization that needs emphasis is that the value of the coupling strength \( \varepsilon \) in OCOMLs should be sufficiently high, while the on-site map parameter \( a \), coupling strength \( \varepsilon \) and drive key \( K_n \) need to be identical for the CMLs under consideration.

### III. Results and Observations

Simulating the cluster structure for two one-way coupled map lattices where one is \( X (i) \) system [as in eq.1] and another is a \( Y (i) \) system [as in eq. 2], we get the following results:

![Fig. 1a](image1.png)

Fig. 1a corresponds to \( X (i) \) system while Fig. 1b corresponds to \( Y (i) \) OCOML system. The two systems consist of 100 different lattice sites (of which 30 are shown above for clarity) being iterated 300 times.

Figure 1 shows two OCOML systems driven by same drive sequence having different initial conditions. Synchronization of spatiotemporal chaos can be visualized by analyzing individual lattice sites of the two systems with same lattice number. Figure 2 shows individual lattice sites in chaotic regime driven by a randomly generated key sequence.

![Fig. 1b](image2.png)

Figure 3 shows the synchronization of lattices shown in figure 2. This plot shows the ratio of amplitudes \((X_n (i)/Y_n (i))\) vs. iteration number for a given lattice site in both the OCOML systems. While figure 3a shows synchronization plot for a randomly generated drive sequence, figure 3b corresponds to synchronization plot for the drive sequence generated from one way coupled ring map lattice system.
Fig. 2 depicts lattice sites \( i = 21 \), for illustration) of the two systems. While Fig. 2a corresponds to a lattice site \( X \) (21), fig. 2b corresponds to lattice site \( Y \) (21), 21 being the lattice number (L).

As soon as the two lattices become synchronized, the ratio of amplitudes \( \frac{X_n(i)}{Y_n(i)} \) becomes equal to one.

Fig. 3a shows synchronization plot of lattices driven by randomly generated common drive sequence. Ratio of amplitude becomes one when lattices synchronize.

Fig. 3b shows the synchronization plot of lattices using common drive sequence generated through coupled ring map lattice \( Z_n(i) \), [as in eq.3].

IV. APPLICATION

The effectiveness of applying spatio-temporal chaos for real time secure communications depends on the time to mask and demask the signal with least complex circuitry.
Fig 4.c

Fig 4.d

Fig. 4a shows a wave sample of an audio file, fig. 4b corresponds to masked version of fig. 4a using $X \,(i)$ lattice site (as shown in Fig. 2a) for masking purpose. Fig. 4c is demasked version of fig. 4b using the corresponding $Y \,(i)$ lattice site (as shown in Fig. 2b) synchronized using random sequence, while fig. 4d is demasked from $Y \,(i)$ lattice using $Z \,(i)$ sequence for synchronization.

The time taken for chaos generation, masking, demasking and synchronization has to be small enough to reduce processing delay so as to prevent any echo. In Fig. 4a we display an audio signal to be transmitted. Fig. 4b shows the signal being masked by the output from a single lattice of the OCOML system. Fig. 4c shows the demasked signal at the receiver end, which faithfully reproduces the original audio signal. The error in fig. 4c and fig. 4d during time (0-20) is due to synchronization error between the two-OCOML systems ($X \,(i)$ & $Y \,(i)$). Note that in this case the synchronization was done using a random number generator as the key. This is faster than running another CML to generate a drive key.

V. CONCLUSION

Lattices of different OCOML systems, starting with different initial conditions have been synchronized using a common drive key. The proposed method eliminates the need for any drive generator, thus minimizing the processing time along with making the system less complex. The efficiency of applying spatio-temporal chaos to real-time communication depends on the time taken by two systems to synchronize and the hardware complexity needed to achieve that synchronization, which has been taken care of by the method discussed in the paper.

VI. REFERENCES