ANTI-PHASE TO IN-PHASE TRANSITION IN COUPLED CHUA’S OSCILLATORS

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Abstract: Experimental observations on the effect of coupling on synchronization of two mutually coupled nonidentical Chua’s oscillators are presented. We observe that as the coupling strength is gradually increased from the very weak coupling, the oscillators move from nonsynchrony to out-of-phase and then to antiphase synchronization. With further increase of the coupling strength, coexisting antiphase and in-phase states are found in the intermediate range. For very strong coupling, in-phase synchronization is observed.

Index Terms: Chua’s oscillator, phase synchronization, antiphase synchronization, out-of-phase synchronization.

I. INTRODUCTION

Studies on synchronization of coupled oscillators are of fundamental importance in the field of nonlinear dynamics [1,2]. Since 1990 [3] many researchers concentrated on synchronization of chaotic systems [4-6] with evidences of different types of correlation between similar variables of interacting oscillators. Of this phase synchronization (PS) [6] is ubiquitous in nature and found to play important roles in many weakly interacting living systems [2]. Biological examples of PS include cardio-respiratory rhythm [7], neural oscillator [8] and cognitive behavior [9].

In the case of PS it is usually found that coupling suppresses the natural frequency mismatch in oscillators and adjust their frequencies to a common locking frequency even though the amplitudes remain almost uncorrelated. Above a critical coupling, the two coupled oscillators develop an \( n:m \) phase locking while the instantaneous phase difference remains bounded, i.e., \( |\phi_1-\phi_2|<\text{constant} \) \( (n, m \) are integers). In the simplest case of 1:1 phase locking, \( |\phi_1-\phi_2|<\text{constant} \) indicates in-phase synchrony. However, other possible phase locking relations as antiphase (with \( |\phi_1-\phi_2|=\pi \)) and out-of-phase synchronization (with \( 0<|\phi_1-\phi_2|<\pi \)) also found. It may be mentioned that antiphase and out-of-phase states lie on the synchronization manifold transverse to the in-phase manifold. Different bifurcation phenomena in transition from antiphase to in-phase synchronization has been elaborated in coupled limit cycle neural oscillators [10], nevertheless a complete understanding of the mechanism of this transition with a turbulent phase for intermediate coupling is yet to be made. Similar turbulent phase has been observed in distributed parametric oscillator [11].

II. EXPERIMENTAL CIRCUIT

 Experimental circuit is shown Fig.1 where each oscillator consists of linear inductor \( L_{1,2} \), capacitors \( C_{1,3} \),

![Fig.1. Coupled Chua’s Oscillator](image)

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\begin{align*}
\frac{dV_{C_{1,3}}}{dt} &= \frac{1}{R_{1,3}C_{1,3}}[(V_{C_{2,4}} - V_{C_{1,3}}) - R_{1,3}f(V_{C_{1,3}})] + \frac{1}{C_{1,3}R_C}(V_{C_{1,3}} - V_{C_{1,3}}) \\
\frac{dV_{C_{2,4}}}{dt} &= \frac{1}{R_{1,3}C_{2,4}}(V_{C_{1,3}} - V_{C_{2,4}} + R_{1,3}I_{L_{1,2}}) \\
\frac{dI_{L_{1,2}}}{dt} &= \frac{1}{L_{1,2}}(-V_{C_{2,4}} - \sigma_{0,1}I_{L_{1,2}})
\end{align*}
\]
The measured voltages $V_{C1,3}, V_{C2,4}$ at corresponding capacitor nodes and the inductor current $I_{1,2}$ are the state variables. The coupling resistance $R_c$ determines the strength of coupling. The slopes $a_{1,2}$ and $b_{1,2}$ of the piecewise linear function are given by

$$a_{1,2} = \left[ \frac{1}{R_{2,9}} - \frac{1}{R_{1,8}} \right] R_{1,8} \quad b_{1,2} = \left[ \frac{1}{R_{4,7}} - \frac{1}{R_{3,12}} \right] R_{3,12}$$

All the circuit components have mismatches since no two similar off-the-shelf components are found identical which is unavoidable in nature. Thus we started with two non-identical Chua's oscillators, where the parameter mismatch induces a natural frequency difference between the oscillators. All components remain fixed throughout this paper except $R_{1,8}$, which is varied to obtain different oscillatory states, periodic to chaotic. The voltages $V_{C1,3}, V_{C2,4}$ at capacitor nodes $C_{1,3}, C_{2,4}$ are measured for different coupling resistance $R_c$ using a digital oscilloscope (HP TDS220, 100MHz) and with 2500 data points in each snapshot. The instantaneous phases $\phi(t)$ of the The instantaneous phases $f(t)$ of the state variables are estimated using Hilbert transform (MATLAB) on the measured time series and the mean frequencies $\Omega(t)$ of the coupled oscillators are estimated as mean rate of change of $\phi(t)$. A simple index of relative phase, $\Delta\Omega/(\Omega_1+\Omega_2)/\Omega_1+\Omega_2$ is taken as a measure [4] of synchronization, which is the frequency difference as percentage of mean frequencies of the coupled oscillators $W(t)$ (i=1,2). Zero difference in mean frequency ($\Delta\Omega = 0$) between the oscillators indicates phase synchronization.

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III. RESULTS

We find that the weakly coupled oscillators move from nonsynchrony to out-of-phase above a coupling threshold and then gradually move to antiphase synchronization with the increase of the coupling strength. With further increase in coupling above another threshold, the coupled oscillators show desynchronization bursts over an intermediate range of coupling strength and finally they move to in-phase synchronization above a large coupling value. In an experiment shown here, the coupled limit cycle oscillators ($R_1=1552 \Omega, R_2=1447 \Omega$) show stable out-of-phase and antiphase above a coupling strength $\epsilon \approx 5.95 \times 10^{-4}$ as shown in Fig.2(a). The corresponding time series of out-of-phase, antiphase and in-phase are shown in Fig.3. Onset of desynchronization starts at a critical coupling $\epsilon \approx 2.8 \times 10^{-5}$ when the frequency difference ($\Delta\Omega$) suddenly jumps high and fluctuates over a range of intermediate coupling strength. With strong coupling, $\Delta\Omega$ decreases and finally disappears at a large coupling threshold ($\epsilon \approx 5.5 \times 10^{-5}$).

We note that the individual frequencies in Fig.2(b) of the oscillators rotate at a common frequency for $\epsilon \approx 5.95 \times 10^{-4}$ although the oscillators remain in either out-of-phase or antiphase. However, individual frequencies increase monotonically until desynchronization starts for $\epsilon \approx 2.8 \times 10^{-5}$. At the onset of desynchronization at $\epsilon \approx 2.8 \times 10^{-5}$, the frequencies of individual oscillators bifurcate again. It is found that the oscillator with higher frequency is less sensitive to coupling as indicated by its slower rate of change than that of the oscillator of lower frequency. This behavior has similarity with the dephasing effect [13] observed in weakly coupled neural oscillators. Alternate cycles of in-phase and antiphase states are observed during the desynchronization bursts in the intermediate coupling range as shown in the time series in Fig.4. The frequency difference $\Delta\Omega$ ($\epsilon$) starts decreasing once again at intermediate coupling and accordingly, the individual frequencies also start coming closer and finally converge at stronger coupling above a coupling $\epsilon \approx 2.8 \times 10^{-5}$ in the in-phase regime as shown in Fig.2.

All three coupling thresholds are found to shift with natural frequency mismatch $[\Delta\omega=\Delta\Omega=0]$ of the uncoupled oscillators, which actually depends upon the parameter set of the individual oscillators. The parameter bifurcation of frequency mismatch ($\Delta\omega$) and coupling ($\epsilon$) is shown in Fig.5 for the three different regions, namely out-of-pase, antiphase and in-phase as obtained from numerical results for a similar situation shown in Fig.6. It shows striking similarities with the behaviors [14] of delayed phase oscillator model.

IV. CONCLUSION

Antiphase, out-of-phase ant in-phase regimes are observed in two Chua’s oscillators coupled diffusively by one variable. Bifurcation of parameter mismatch and coupling is obtained numerically. Interesting features of
coexisting in-phase and antiphase states are observed for intermediate range of coupling.

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V. REFERENCES


Fig.2. Anti-phase to in-phase transition. Plotted are the frequency difference $\Delta \Omega(\varepsilon)$ in the upper plot and the individual frequencies $\Omega_i(\varepsilon)$ in lower plot as a function of coupling $\varepsilon$ for the experiment. Both oscillators are parameterized in the period-1 and period-2 regime, respectively, with the following parameter values. $R_1=1552\Omega$, $R_2=1447\Omega$ with frequency mismatch $\Delta \omega=239\text{Hz}$.

Fig.3. Time series of the voltages of the oscillators respectively. Different synchronization regimes for increasing coupling strength from top to bottom. Top row: out-of-phase for coupling just after the first onset of phase synchronization; middle row: anti-phase; bottom row: in-phase for large coupling values after the second onset of phase synchronization. Parameter values, $\varepsilon = 5.95\text{E-6}$ (top), $\varepsilon = 1.28\text{E-5}$ (middle), $\varepsilon = 5.55\text{E-5}$ (bottom).
Fig. 4. Time series of VC1, VC3 for intermediate coupling $R_c=30.5$ m$\Omega$ [and $R_1=1552$ $\Omega$, $R_2=1447\Omega$].

Fig. 5. Two parameter bifurcation of $\varepsilon$ and $\Delta \omega$: Open circle for in-phase, bold square for antiphase and solid circle for out-of-phase thresholds.

Fig. 6. Anti-phase to in-phase transition. Plotted are the frequency difference $\Delta \Omega(\varepsilon)$ (top row) and the individual frequencies $\Omega_{1,2}(\varepsilon)$ (bottom row) as a function of coupling $\varepsilon$ for numerical simulation. Oscillators are parameterized in the period-1 and period-2 regime, with the following parameter values. $R_1=1780\Omega$, $R_2=1690\Omega$ with frequency mismatch $\Delta \omega=75.39$ Hz. The two oscillators show enhanced transition to out-of-phase synchronization ($0<\Delta \phi<\pi$) for coupling $\varepsilon=5.95E-6$, and antiphase synchronization ($\Delta \phi=\pi$) for $\varepsilon=1.28E-5$ just before the onset of large desynchronization, and finally in-phase synchronization ($\Delta \phi=0$) for large coupling strength $\varepsilon>5.55E-5$. 