The Role of Host Migration on Host-Parasite Population Dynamics

Ganesh Bagler and Somdatta Sinha

Abstract— Most organisms in nature exist in a trophic network. Their population sizes are dependent on the activity and abundance of the other interacting species. Their survival, extinction and population dynamics also depend on environmental and demographic parameters, such as growth rates, interaction strengths, migration, etc. Here we study the dynamics of a discrete host-parasite population under free growth, and when the host undergoes constant migration. We show, both analytically and numerically, that migration affects the dynamics of the host and parasite in a non-intuitive manner. The population dynamics of the host and parasite is stabilised, irrespective of its intrinsic growth rate, in presence of small immigration rate. This result may aid in designing intervention strategies in infectious disease scenario.

Keywords— Exponential Map, Migration, Host-Parasite Model, Population Dynamics, Chaos.

I. INTRODUCTION

POPULATIONS in nature experience different types of external perturbations from environmental changes and demographic events that regulate their abundance and dynamics. In interacting populations such as, the host-parasite system, migration of hosts, harvesting, culling, vaccination, quarantine, and segregation type of events are commonly used to control population abundance. There has been much work on the role of migration on population dynamics, and several parasitic diseases have been modelled to describe and predict the spatiotemporal spread of host-parasite populations [1]. Hosts tend to move away when an infectious disease breaks out. A common method for limiting an epidemic is also done by segregation of hosts, so that parasites can not attack the uninfected ones. This paper attempts a theoretical study of the response of the host and parasite population dynamics to host migration or host segregation.

We first study the discrete host-parasite dynamics when the host grows according to the realistic density-dependent exponential model, and then study how the population dynamics is affected, when hosts undergo a fixed amount of migration, or are segregated by a fixed amount every generation. Our analytical and numerical results show that the host-parasite dynamics under free growth shows both stable and unstable dynamics (including quasi-periodicity and chaos) for different values of the demographic parameters, such as the intrinsic growth rate of the host, and the infectivity of the parasite. Our analysis shows that migration/segregation of host can induce changes in dynamics of both host and parasite populations. A non-intuitive result is that an unstable population is stabilised with increasing addition of hosts. Stabilising the dynamics can aid in application of intervention strategies such as vaccination. Thus, our results show that it does not help in controlling the disease effectively by emigration and segregation of the host population.

II. DISCRETE POPULATION GROWTH MODELS

To assess the effect of the inter-specific interactions and migration on the dynamics of populations of the host and parasite, we present the discrete population dynamic models as follows. First, we state the single population growth model for the host, and show its dynamics with changes in growth parameter. Then the interacting population model of the host and parasite (HP model) is given where parasites grow on hosts to regulate their size. Here, both the growth rate of the host and the infectivity parameter of the parasite play important roles in regulating population sizes. We then state the results of the HP model when the host undergoes a fixed amount of migration at every generation.

A. Exponential Model for Host Growth

The Exponential model for the discrete host population is given by

\[ X_{t+1} = F(X_t) = X_t e^{r(1-X_t)} \]  

where \( X_{t+1} \) & \( X_t \) represent the population sizes at any two consecutive generations \( t \) and \( t+1 \); \( F \) is a non-linear density dependent growth function, which is controlled by a single parameter, the intrinsic growth rate \( r \), that plays a role in determining the size of the population in the next generation. This form of \( F \) has a convex “single-hump” shape. There are many other formulations of \( F \) in discrete single population models, which have similar shape decided by one or more parameters. All these models exhibit a variety of dynamics – from equilibrium to periodic, and period doubling bifurcations leading to chaos, with increasing growth rate \( r \) [2]. Fig. 1 shows the bifurcation diagram for the Exponential model for different values of \( r \).

Fig. 1. Bifurcation diagram for the Exponential Model with increasing “r”.

---

Ganesh Bagler and Somdatta Sinha (sinha@ccnb.res.in) are with Centre for Cellular & Molecular Biology, Uppal Road, Hyderabad. GB thanks CSIR for Senior Research Fellowship.
B. The Host-Parasite (HP) Model

The Host-Parasite (HP) model system [3] is described by

\[ \begin{align*}
H_{n+1} &= H_n e^{r(1-H_n)} e^{-\beta P_n}, \\
P_{n+1} &= H_n (1 - e^{-\beta P_n}),
\end{align*} \]

where \( H_n \) and \( P_n \) are the host and parasite populations at \((n+1)^{th}\) and \(n^{th}\) generation respectively; \( r \) is the intrinsic growth rate of host; and, \( \beta \) is the searching efficiency of the parasite indicating its infectivity. The parasite can grow only in the presence of the host, and the effect of parasite is to reduce the size of the host population. The Host-Parasite interaction is through the independent and random search by the parasite with constant searching efficiency. This interaction function is assumed to be a Poisson process, and is given by \( e^{-\beta P_n} \) [4].

Fig. 2 shows the bifurcation diagram of the host in the HP model for different values of \( \beta \). The plot shows that an intrinsically chaotic host population following Exponential growth model \((r = 3.0; \text{see Fig. 1})\) is stabilised by the action of the parasite at medium values of \( \beta \). But both host, as well as parasite populations, show quasi-periodic and chaotic dynamics as \( \beta \) is increased. This dynamic behaviour of the host population to parasite infection is observed for a large range of host growth rate \( r \). Details of this phenomena is shown elsewhere [5]. Thus, the effect of the interaction between the host and parasite can influence both abundance and dynamics of the host population.

![Bifurcation diagram for the HP system for \( r = 3.0 \) for increasing \( \beta \).](image)

C. The HP Model Under Fixed Migration of the Host

The Host-Parasite (HP) model system under migration of the host is described by

\[ \begin{align*}
H_{n+1} &= H_n e^{r(1-H_n)} e^{-\beta P_n} + L \\
P_{n+1} &= H_n (1 - e^{-\beta P_n}),
\end{align*} \]  

where \( L \) is a parameter describing the constant rate of migration of the host at every generation, which can assume any real value. When there is no migration, \( L = 0 \). \( L < 0 \) for emigration, and \( L > 0 \) for immigration.

The steady states for the HP system in (3) is obtained from

\[ \exp \left[ -\beta H^* \left( 1 - \frac{H^* - L}{H^* e^{r(1-H^*)}} \right) \right] = \frac{H^* - L}{H^* e^{r(1-H^*)}} \]

(4)

\[ P^* = H^* \left[ 1 - \frac{H^* - L}{H^* e^{r(1-H^*)}} \right]. \]

(5)

To solve for \( H^* \), we consider the following transcendental equation

\[ 1 - \exp \left[ -\beta H^* \left( 1 - \frac{H^* - L}{H^* e^{r(1-H^*)}} \right) \right] = 1 - \frac{H^* - L}{H^* e^{r(1-H^*)}} \]

(6)

The solution of (6) gives the steady state values \( H^* \), and then using (5) we can find the corresponding \( P^* \).

We use a graphical approach to solve for \( H^* \) for different values of \( L \). Let us denote

\[ f = 1 - \frac{H^* - L}{H^* e^{r(1-H^*)}}, \]

(7)

\[ g = 1 - \exp \left[ -\beta H^* \left( 1 - \frac{H^* - L}{H^* e^{r(1-H^*)}} \right) \right]. \]

(8)

For a particular value of \( L \), when we plot \( f \) and \( g \) for different values of \( H \), the intersections of these two curves satisfy (6) at \( H = H^* \). Using (5) we can find the corresponding \( P^* \).

Fig. 3 (a, b) shows, for \( r = 3 \), the \( f, g \) plots for (a) \( \beta = 3.5 \), \( L = 0.13 \), and, (b) \( \beta = 4.5 \), \( L = 0.26 \). The plots show that (6) gives two values of \( H^* \). Hence there are two steady states \((H^*, P^*)\) of the Host-Parasite system. Since we are interested in biological populations, we restrict the discussion only to the positive steady states, i.e., when both \( H^*, P^* > 0 \). This is true only for the first \( H^* \) value. Thus we are interested in the stability of the first steady state only.

![The f–g plots of the HP system: (a) \( \beta = 3.5 \), \( L = 0.13 \), and (b) \( \beta = 4.5 \), \( L = 0.26 \). The corresponding stability plots: (c) \( \beta = 3.5 \), and (d) \( \beta = 4.5 \).](image)

Linear stability analysis of the positive steady state shows that the HP system will be stable if they satisfy the condition

\[ 2 > 1 + C > |B| \]

(9)

where,

\[ C = \beta e^{r(1-H^*)} \left( \frac{H^* - L}{e^{r(1-H^*)}} \right) \left[ 1 - r \left( \frac{H^* - L}{e^{r(1-H^*)}} \right) \right], \]

and
\[ B = \left[ (1 - rH^*)e^{r(1-H^*)} + \beta H^* \right] \left( \frac{H^* - L}{e^{r(1-H^*)}} \right). \]

Because of the presence of the transcendental function, none of the relations obtained above can be simplified further.

We compute \( B \) and \( C \) numerically to check the local stability for the corresponding steady states. Fig. 3 (c, d) show the \(|B|\) and \(1 + C\) values plotted for (c) \( \beta = 3.5 \) and (d) \( \beta = 4.5 \), for different values of migration \((L)\), at \( r = 3 \). The plots show that the condition of local stability, as given in (9), is satisfied for \( L > 0.12 \) for \( \beta = 3.5 \), and for \( L > 0.26 \) for \( \beta = 4.5 \). Thus from the linear stability analysis we can obtain an estimate of the effect of host migration on the stability of the HP populations for different \( r \) and \( \beta \).

**D. Dynamics of the Host-Parasite Model Under Migration of the Host**

To study the dynamics of the host and parasite populations for changes in the parameters \( r \), \( \beta \), and \( L \), we performed computational studies of (3). The bifurcation diagrams in Fig. 4 and Fig. 5, obtained numerically, show the effect of host migration on the complex dynamics exhibited by the HP model for \( r = 3 \) and \( \beta = 3.5 \) and \( \beta = 4.5 \) respectively, for increasing immigration and emigration. The dynamics of the freely growing HP system \((L = 0)\) for parameters \( r = 3 \) and \( \beta = 3.5 \) and \( \beta = 4.5 \) are quasi-periodic and chaotic (see Fig. 2). Fig. 4 and Fig. 5 show that both the types of dynamics are stabilised to a fixed point state with increasing immigration \((L > 0)\).

Increasing amounts of emigration \((L < 0)\) also control the chaotic dynamics of the populations to higher periodic dynamics for \( \beta = 3.5 \) as shown in Fig. 4. But both figures show that higher rates of emigration drive the populations to extinction. A chaotic HP population (refer Fig. 5) can support very small host-parasite population. The bifurcation diagrams also show that emigration can control the chaotic dynamics of the populations to higher periodic dynamics for some values of \( \beta \), but any higher rate of emigration drives the populations to extinction. The result that immigration stabilises the dynamics indicates that it may not help in containment of infection by segregating the host population, as it is easier to implement intervention measures in a population at steady state.

We have shown [6], [7], [8] elsewhere, using coupled map lattice approach, that it is also possible to modulate the dynamics of the HP system in a large meta-population under constant migration of the hosts or parasites to the neighbouring sub-populations. Thus the dynamics of interacting species populations can be modulated by demographic and ecological processes, such as migration, both in a single and in meta-population.

![Fig. 4. Bifurcation diagram for HP system with migration of host for \( r = 3 \) and \( \beta = 3.5 \).](image1)

![Fig. 5. Bifurcation diagram for HP system with migration of host for \( r = 3 \) and \( \beta = 4.5 \).](image2)

**REFERENCES**


